Example

Consider the following two parametric curves:

$$\begin{cases} x = 2-2t \\ y = 3+t^2 \end{cases} \qquad \begin{cases} x = 2t-4 \\ y = t^2 \end{cases}$$

Do these curves intersect? If so, where?

Solution attempt

$$\begin{cases} 2-2t = 2t-4 \\ 3+t^2 = t^2 \end{cases}$$

This has no solutions...

⚠ This system solves for a collision rather than an intersection.

same place (not necessarily same time)

same place & same time

Solution

$$\begin{cases} 2-2t = 2s-4 \\ 3+t^2 = s^2 \end{cases}$$

Rearranging first eq: $t = 3-s$

Plug into second: $3+(9-6s+s^2) = s^2$

$12 - 6s = 0$

$s = 2$ and $t = 1$

<u>Check</u>: $2 - 2t = 0$ $\quad$ $2s - 4 = 0$

$3 + t^2 = 4$ $\quad$ $s^2 = 4$

Yes, the curves do intersect @ the point $(0, 4)$.

<u>Food for thought</u>: How would you find a point of self-intersection? e.g. if you had a parametric curve like 

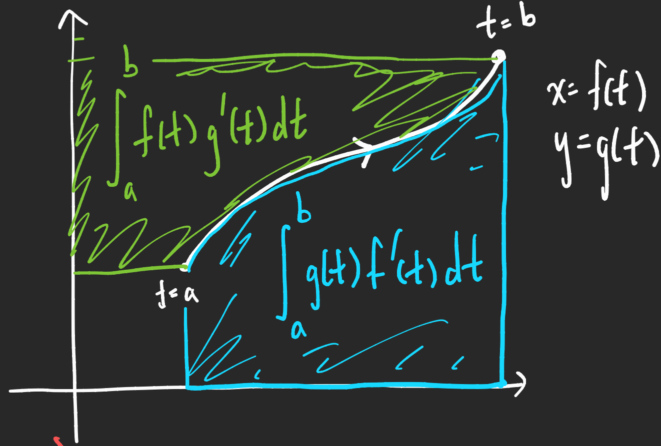
Rmk for dealing w/ $\sin^2 t$, $\cos^2 t$ in integrals:

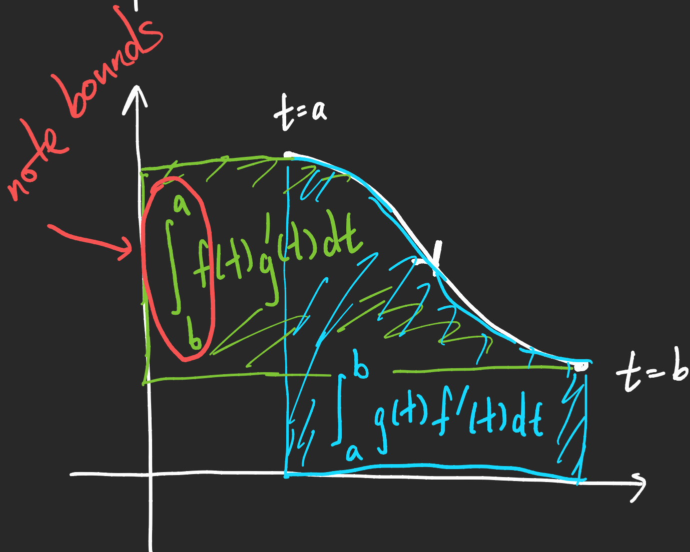
$$\cos(2t) = \cos^2 t - \sin^2 t$$
$$= 1 - 2\sin^2 t$$
$$= 2\cos^2 t - 1$$

Rearrange: $\cos^2 t = \dfrac{1 + \cos(2t)}{2}$

$$\sin^2 t = \frac{1 - \cos(2t)}{2}$$

Some comments on area:

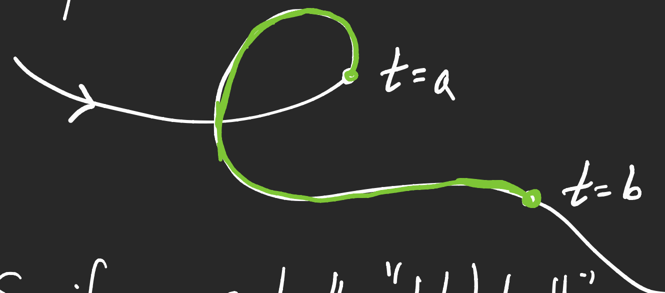




Some comments on arclength

$$\boxed{\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt}$$

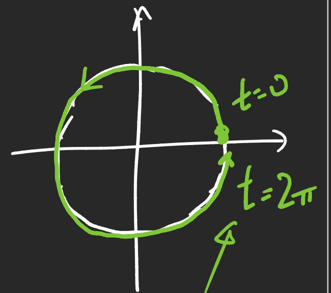
computes the distance traveled by the particle between times $t=a$ and $t=b$.



So, if you want the "total length" of a curve, you should determine what parameter bounds cause the particle to trace out the curve exactly once.

ex) $x = \cos t$

$y = \sin t$



§10.2 51·52: ask for distance traveled as a function of time t

$$\int_0^t \sqrt{\left((2\sin u)\cos u\right)^2 + \left((2\cos u)(-\sin u)\right)^2}\, du$$

is the distance traveled between time 0 and time t for #51, for example.